Matter: Very Simple

Energy of Individual Particles

All materials are made up of particles, and particles have energy. The particles of an ideal gas have only kinetic energy since forces between the particles are negligible. However, this cannot be true of a solid or a liquid or the particles would not stay in the phase, and the particles in real gases exert both attractive and repulsive forces on each other during collisions.

Imagine a collision between two particles in a real gas:

- As the particles approach, the attractive forces cause an increase in the speed and kinetic energy.
- The particles then get closer and the speed decreases, being momentarily zero before they start to separate.
- The process is then reversed.

There is an interchange between potential energy and kinetic energy during the collision. At first the particles speed up due to short-range attractive forces; then they slow down due to close-range repulsive forces. There is an equilibrium in between.

The particles of real gases, like those of solids and liquids, have both potential and kinetic energy. The distribution of energy between kinetic and potential is constantly changing and so is said to be random.

The total amount of energy of the particles in an object is known as its **internal energy**. At constant temperature the internal energy of an object remains unchanged, but the contributions of individual particles to that energy change due to the transfer of energy during interactions between particles.

Measuring Temperature

The Celsius scale is based on the ice and steam-points of water. However,

- thermometers which use different thermometric properties only agree at the points of calibration,
- the Celsius scale is not an absolute scale, so 20°C is not twice as hot as 10°C.

Hence we use the scale Kelvin, which is based on the triple point of water (the temperature and pressure at which all three of its phases coexist in thermodynamic equilibrium).

- The zero point corresponds to the minimum internal energy of the particles,
- there is a unique value for any temperature, which does not depend on the instrument being used to measure it,
- 20K is twice as hot as 10K.

To convert from Celsius to Kelvin, we add 273.15.

Specific Heat Capacity

Changing the temperature of an object involves a change in its internal energy, the total potential and kinetic energy of the particles.

The energy transfer required to change the temperature of an object depends on:

- the temperature change,
- the mass of the object,

- the material the object is made from.

These are all taken into consideration in the concept of **specific heat capacity**. The term 'specific' means 'for each kilogram'.

The specific heat capacity of a material, c, is defined as the energy transfer required to raise the temperature of 1kg of the material by 1°C.

$$\Delta E = mc\Delta\theta$$

Where ΔE is the change in energy (J), c is the specific heat capacity (J kg^{-1} K^{-1}), m is the mass (kg) and Δθ is the change in temperature (K or °C).

Changing Phase

If you try to squash a solid or liquid, the particles are pushed closer together and they repel each other. Stretching has the opposite effect; when the separation of the particles is increased the forces are attractive. At increased separations the particles have increased potential energy and this has to be supplied for a process that involves expansion to take place.

When a substance changes **phase** there is a change in the potential energy of the particles. For most substances there is a small increase in potential energy when changing from solid to liquid and a much bigger change from liquid to gas.

An Ideal Gas

The concept of an **ideal gas** is useful because the behaviour of all gases is close to ideal provided that the gas is neither at a high pressure not close to its boiling point.

Charles' Law: $V \propto T \Rightarrow V/T = \text{constant}.$

(Volume of fixed mass at constant pressure).

Boyles' Law: $p \propto 1/V \Rightarrow pV = \text{constant}.$

(Volume of fixed mass at constant temperature).

Pressure Law: $p \propto T \Rightarrow p/T = \text{constant}.$

(Volume stays constant).

Where V is the volume (m^3), T is the absolute temperature (K), and p is the pressure (Pa). Gases which obey Boyle's Law are called **Ideal Gases**.

Combined Gas Laws:

$$p_1V_1 / T_1 = p_2V_2 / T_2$$

pV= nRT = NkT

(Mass is constant)

Where n is the number of **moles**, R is the **universal gas constant, 8.31 $JK^{-1}mol^{-1}$**, N is the number of **particles** and k is the **Boltzmann Constant**, **1.38×10^{-23} JK^{-1}**.

The number of particles in a mole is given by the **Avogadro Constant**, **$N_A = 6.02 \times 10^{23}$ mol^{-1}.**

N.B. $k \times N_A = R$; 1.38×10^{-23} $JK^{-1} \times 6.02 \times 10^{23}$ mol^{-1} = 8.31 $JK^{-1}mol^{-1}$

Kinetic Theory Equation

$$pV = \frac{1}{3} Nm < c^2 > ¿$$

(1/3 because there are 3 Cartesian directions: x, y, z)

Where p = pressure (Pa), V = volume (m^3), N = number particles, m = mass (kg), and $<c^2>$ = **mean squared speed** of the molecules (ms^{-1}).

The mean squared speed, $<c^2> = \frac{\sum c^2}{N}$, **root mean squared, $c_{r.m.s}$** = $\sqrt{(\frac{\sum c^2}{N})}$

Where c is the speed of the particles and N is the number of them.

Note also that: $\rho = \frac{Nm}{V}$ (because it's the total mass of all the molecules / Volume), and hence:

$$p = \frac{1}{3} \rho < c^2 > ¿$$

Where ρ is the density of the gas (kgm^{-3}).

The mass of an electron, proton and neutron respectively are given as:

$m_e = 9.11 \times 10^{-31}$ kg
$m_p = 1.673 \times 10^{-27}$ kg
$m_n = 1.675 \times 10^{-27}$ kg

The average translational kinetic energy of any kind of molecule in an ideal gas is given by:

KE_{avg} = $\frac{3}{2} kT$ **(per molecule)**

KE_{avg} = $\frac{3}{2} RT$ **(per mole)**

Force = rate of change of momentum, $F = \frac{(\Delta mv)}{t}$

Pressure = Force/ Area, $p = \frac{F}{A}$

Density = Mass/ Volume, $\rho = \frac{m}{V}$

Upthrust/buoyancy thrust/down thrust (N) = weight of air displaced (kg)
Weight it can lift = up thrust – weight of balloon etc.

Force = Density × volume× g, $F = \rho \times V \times g$ (i.e. F = mg)
where g = 9.81 Nkg^{-1}.

Gas particles travel, on average, at around $200ms^{-1}$.

- Particles move in random motion because they are continually being subjected to collisions.
- When particles collide with objects, there is no change in kinetic energy, hence the collisions are **elastic** (there is KE transfer, but it remains constant). Momentum and total energy are conserved.
- Pressure increases as volume decreases because of more frequent collisions with the walls.
- Pressure increases with temperature because the higher KE results in more collisions (at constant volume).

- Pressure of a gas is due to lots of molecular impacts. Each impact of a molecule with a solid surface exerts a tiny force on the surface. Lots of impacts all over the surface average out to give a steady force on the surface; and hence a steady pressure.
- The molecule does not lose any KE and hence speed as it hits the surface of the wall. But it's momentum and velocity change due to the change in direction.
- To change momentum, a force is needed. Hence we can use this to give an expression for the force. But the following assumptions must be made:

Point molecules – avoids need to consider sizes of different types of molecules. If the volume of the space is much greater than the volume of the molecules, the molecules are effectively points.

No intermolecular attractions – if they are significant, impacts on solids are reduced in force. The molecules away from the surface hold back any molecules towards the surface so the impact force is reduced. Provided the gas is not at high density or low temperature, the assumption is valid.

Random Motion – the average of many random impacts gives a smooth pressure.

Elastic Collisions – if the collisions were inelastic, the KE of the molecules would be converted into other form of energy. The gas pressure would hence fall, and since this does not happen, the collisions must be elastic.

Impact Time – force = change in momentum / time taken. Provided the impact time is much shorter than the time between impacts, then change in momentum / time between impacts gives the average force.

From this we can derive the kinetic theory equation.

For N point molecules, each of mass m, the total kinetic energy is given by adding the KE of all N molecules. If the molecular speeds are $c_1, c_2, ... c_N$, then the molecular KE's are ½ mc_1^2, ½ mc_2^2,... ½

mc_N^2. So the total KE = ½ Nm$\langle c^2 \rangle$.

Explanation of the ideal gas equation (pV = nRT) is now possible if we bring temperature into it. The Kinetic Theory equation is pV = 1/3.Nm$\langle c^2 \rangle$. If we assume that Nm$\langle c^2 \rangle$ is proportional to the absolute temperature T (i.e. Nm$\langle c^2 \rangle$ = constant × T), then the kinetic theory equation becomes pV = constant × T. Which is almost the same as the ideal gas equation. We can make it exactly the same by assuming 1/3.Nm$\langle c^2 \rangle$ = nRT. What does this assumption mean? The total KE of N gas molecules is ½ Nm$\langle c^2 \rangle$ which is therefore 3nRT/2. So the total KE is proportional to the absolute temperature.

The total KE of N molecules ½ Nm$\langle c^2 \rangle$ = 3nRT/2.
The mean KE of a gas molecule = total KE / no. molecules:

$$\frac{1}{2} Nm \frac{[c^2]}{N} = \tfrac{1}{2} m\langle c^2 \rangle = \frac{3nRT/2}{N}$$

For n moles of gas, the total number of molecules N = n × N_A where N_A is Avogadro's Constant, and hence:

Mean KE of gas molecule = 3/2 kT, where k is Boltzmann Constant.

In effect, we have 'proved' the ideal gas equation pV = nRT from the kinetic theory equation pV = 1/3.Nm$\langle c^2 \rangle$ by assuming the mean KE of a gas molecule ½ m$\langle c^2 \rangle$ = 3/2 kT. The ideal gas equation is based on experiments. The kinetic theory has been used above to derive the ideal gas equation. For a gas which does not obey the ideal gas equation, we can look at the assumptions to see why they do not apply. Avogadro's Hypothesis can also be explained using the kinetic theory equation.

Archimedes' Principle

The apparent loss of weight when a body is supported in fluid equals the weight of the fluid which has been displaced. i.e. the buoyancy or upthrust acting.

e.g. The volume of gas in an airship is $10^5 m^3$. What load can it lift in addition to the weight of the gas when filled with a) Hydrogen b) Helium?
Density of air = $1.2 kgm^{-3}$; density of hydrogen = $0.09 kgm^{-3}$; density of helium = $0.18 kgm^{-3}$; g = 9.8 Nkg^{-1}.

a) $0.09 \times 10^5 = 9000\, kg$; $Upthrust = 1.2 \times 10^5 = 120000\, kg$
$Displacement = (120000 - 9000) \times 9.8 = 1.1 \times 10^6\, N$

b) $0.18 \times 10^5 = 18000\, kg$
$(120000 - 18000) \times 9.8 = 1.0 \times 10^6\, N$

e.g. An advertising balloon, filled with helium, is tied to the ground so that it hovers above a shopping centre, the volume of the balloon is $25m^3$ and the mass of the balloon itself without the gas is 15kg. Calculate the tension in the tethering rope on a still day.

$25 \times 0.18 = 4.5\, kg$; $25 \times 1.2 = 30\, kg$; $15 \times 9.8 = 147\, N\ (weight\ acting\ down)$

$Upthrust = (30 - 4.5) \times 9.8 = 249.9\, N$

$Tension = 249.9 - 147 = 103\, N$