Out Into Space

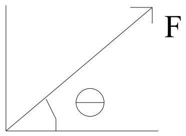
Quick Revision:

Kinematic Equations:

- $v = u + at$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2} at^2$

Where v = final velocity, a = acceleration, u = initial velocity, t = time, s =distance.

Resolving Vectors - Expressing as a direction in x and y (used in projectile motion)



$x = F\cos\theta; y = F\sin\theta$

Newton's First Law/Law of Inertia/Galileo's Principle: *When no force acts on an object (or when the forces acting on it cancel), it moves in a straight line at constant speed.*

Newton's Second Law: *The acceleration of an object equals the total force acting on it, divided by its (constant) mass, or F = ma.*

Newton's Third Law: *For every force, there is a reaction force, equal in magnitude and in the opposite direction. Momentum is conserved, i.e., momentum cannot be created or destroyed, but only transferred from one object to another.*

Kepler's Laws - 16th Century Europe

It was thought the planetary motions were perfectly circular. Kepler discovered the paths were elliptical.

Eccentric ellipse:

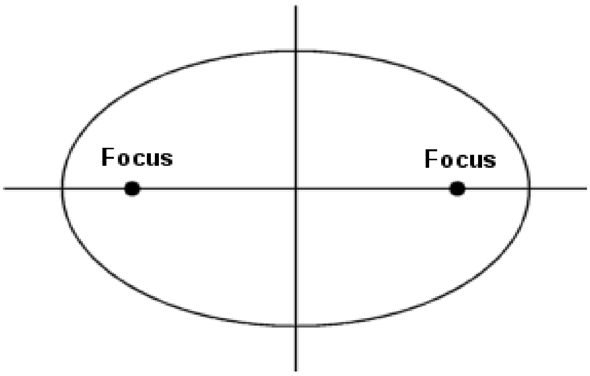


Figure 1 – Showing the foci of an ellipse.

Ellipses are given eccentricity numbers, e. As the ellipse becomes less eccentric, the foci become closer. A perfect circle has the focus in the centre, i.e. e = 0. The higher the value of e, the more elliptical it becomes. One of the foci in out solar system is the sun.

Kepler's First Law

- The orbit of a planet about a star is an ellipse with the star at one focus.

Kepler's Second Law

- Orbiting bodies (elliptical and circular) map out equal areas in equal periods of time. (Also known as the **law of equal areas**).

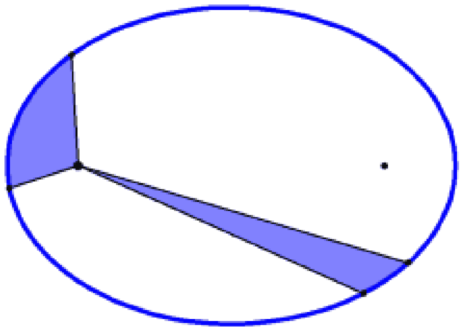


Figure 2 – Representing Kepler's Second Law.

Kepler's Third Law

- $\frac{T^2}{R^3} = constant$ this constant varies depending on the orbiting body.

Where T = orbital time and R = average distance from the centre of the orbit.

This constant will differ for each planet, but since the mass is not involved, the moon of a planet and an orbiting spaceship, for example, will give the same constant value, since they are both orbiting the same planet.

Circular Motion

e.g.: Orbiting planets/satellites (gravity), conker on string (tension in the string), swings/pendulums (tension in chain etc.), roundabouts (friction), fairground rides – e.g. “centrifuge” (the wall behind, pushing back), cars going round corners (friction between tires and road), wheels (tension in rubber) etc.

In *all* cases of circular motion there is a force involved which keeps the mass in a circular orbit. This force is called a **centripetal force**. This force is always towards the centre of rotation, and is due to the body's inertia. (Newton's first law of motion).

$Acceleration = \frac{(\Delta v)}{(\Delta t)}$, v = vector quantity

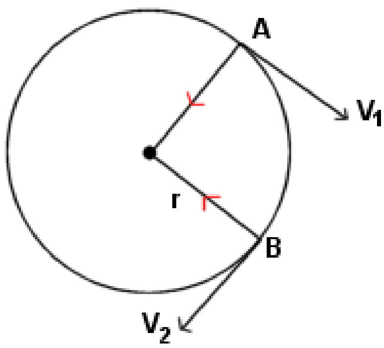


Figure 3 – Centripetal Acceleration

The speed is constant, but the velocity changes due to direction change. Hence there must be an acceleration. Since acceleration is always in the direction of the force, there is an **acceleration towards the centre** of the circle.

An orbiting spacecraft appears to be weightless because it is in free fall (like in an aeroplane when training astronauts). Astronauts appear to be weightless, not because of the lack of gravity, but because they are in free fall – this is due to centripetal acceleration.

F = ma

$$Centripetal\ Acceleration = \frac{v^2}{r} = r\,\omega^2$$

$$Centripetal\ Force = \frac{mv^2}{r}$$

Where v = velocity or orbiting body, r = radius of circle (distance from centre of circle to centre of orbiting body), m = mass of orbiting body, and ω is the angular velocity (rate of change of displacement with time)

Gravitational Fields

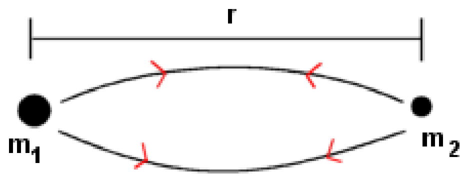


Figure 4 – Gravitational attraction between two masses

Newton's law of universal gravitation states the following:

> *Every object in the Universe attracts every other object with a force directed along the line of centres of mass for the two objects that is proportional to the product of their masses and inversely proportional to the square of the separation between the two objects.*

$$F \propto \frac{(m_1 m_2)}{r^2} \quad \text{hence} \quad F = \frac{(G m_1 m_2)}{r^2}$$

where G is the **universal gravitational constant = 6.67 × 10^{-11} Nm^2kg^{-2}.**

Field strength is the same as acceleration and is determined by e.g. The mass of the Earth (for gravity) and not the mass of the person experiencing "the fall".

F = ma, F = mg.

For a massive object like the Earth,

$$F = \frac{GMm}{r^2}$$

where M = larger mass, m = smaller mass

Because F = mg, $mg = \frac{GMm}{r^2}$ and hence field strength, $g = \frac{GM}{r^2}$

Gravitational fields are always attractive (except for black energy!). Figure 5 shows a radial field – everything is attracted towards the centre. The gravitational strength at centre = 0 (mass gives gravity and the mass surrounds the centre equally, hence the net forces sum to zero). The strongest pull from gravity is on the surface, which for Earth is 9.81 Nkg^{-1}. If we dig a hole through the centre of the Earth and ignore external forces, other than gravity, the mass we drop down will oscillate sinusoidally.

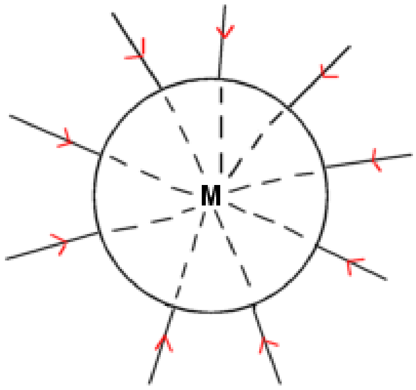


Figure 5 – A Radial Field (lines are meant to be straight!)

Attractive fields are always given a negative sign. This is to overcome the gravitational potential energy problem so it is not infinite, hence

$$F = \frac{-GMm}{r^2}$$

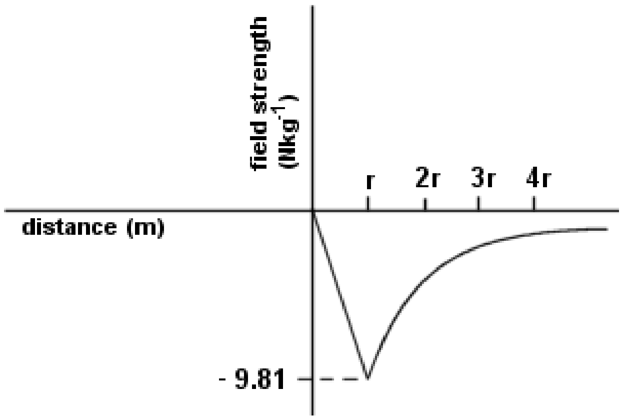


Figure 6 – Graph showing g against r, on Earth

$g \propto \frac{1}{r^2}$ Because -9.81 is the maximum field strength at 1 radius (i.e. on the surface of the Earth). Origin of the graph is the centre of the Earth (zero field strength).

A Trip to the Moon

Gravity curves space-time. These curves are called **potential wells**, and are formed due to a massive object (analogous to a trampoline – see figure 7).

A space shuttles engines need to be on to escape Earth's gravity, but then they coast to the moon. Shuttles do not need to use engines to get back to the Earth from the moon because it is “downhill”

- down towards the "deeper" well.

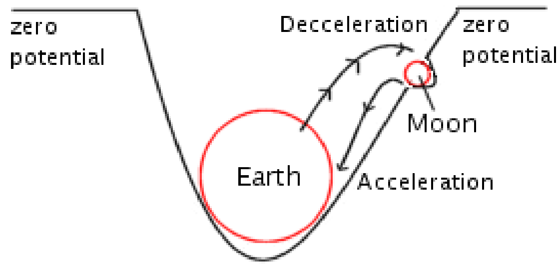


Figure 7 – Potential Wells and curved spacetime.

$F = \frac{GMm}{r^2}$ g = acceleration = $\Delta \frac{v}{t}$

Regular readings were taken of the velocity of the shuttle – this allowed the value of 'g' to be calculated.

Uniform and Non-Uniform Fields

Close to the Earth's surface there is a uniform field (force is constant at all heights within this area). Obviously there is a slight change, but this is negligible. As we get further from Earth, the potential energy increases.

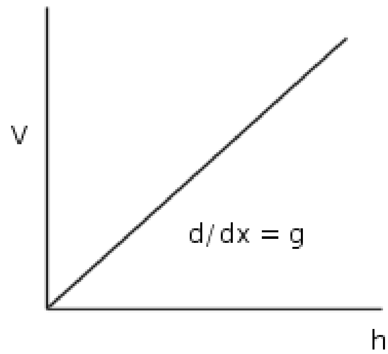
Gravitational potential, V or V_G, is gravitational potential energy per kilogram.

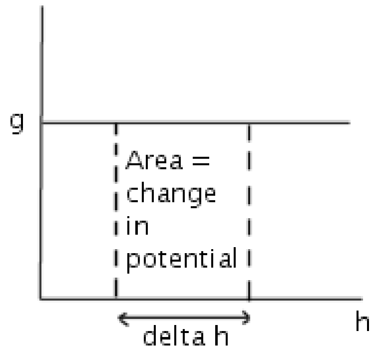
$$V = \frac{mgh}{m} = gh$$

Change in potential $\Delta V = g \Delta h$

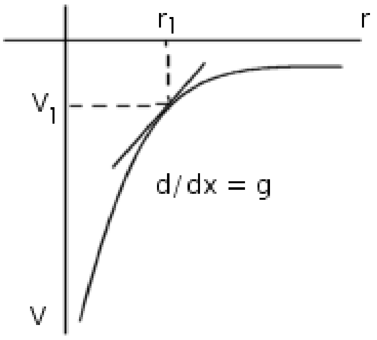
- Plot a graph of potential (V) against displacement and the gradient is equal to the gravitational field strength 'g'. (V = gh)
- Plotting field strength against displacement gives the area under the graph as change in potential ($\Delta V = g \Delta h$).
    - This allows you to calculate the change in potential energy, as you know the mass of the spaceship etc.

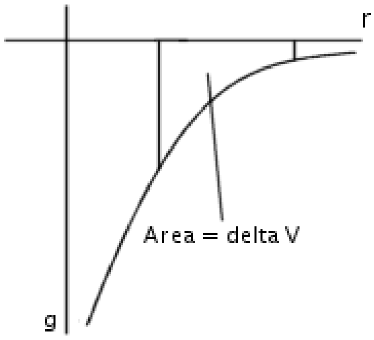
Uniform Fields:





Non-Uniform Fields:





The potential varies as $\frac{1}{r}$.

Where spacetime is flat there is zero potential. The more massive the object, the deeper the potential well – hence the harder it is to 'get away' (more gravity).

Potential is always negative, or zero. In a non-uniform field, we increase the potential energy until we get zero potential. Infinity is the distance far enough away for the gravity to become negligible.

Potential in a non-uniform field, $V = \frac{-GM}{r}$ where G = universal gravitational constant.

Escape Velocity

Escape velocity is independent of mass. Escape velocity is the velocity needed to get to zero potential – away from the force of the large body, and not within its orbit.

Total energy = KE + PE
Gain in KE = Loss in PE (PE = $V_G \times M$)

$$\tfrac{1}{2} m v^2 = \frac{-GMm}{r}$$

$$v = \sqrt{\left(\frac{2GM}{r}\right)}$$ This is the escape velocity.

Black holes are any objects which are so massive, their potential wells are so deep that not even light can escape their gravitational attraction – i.e. min escape velocity = 3×10^8 ms^{-1} (nothing is FTL).

Geostationary orbit means it goes around the Earth once every 24 hours. A transfer orbit can also be called a Hohmann orbit.

Momentum

Momentum can be calculated by:

$$p\ (kgms^{-1}) = mv$$

Changes of momentum require a force. The change of momentum depends upon how long you apply the force.

$$Ft = mv - mu$$

Where u = initial velocity, v = final velocity. Hence Force × time = change in momentum.

Ft is sometimes referred to as the **impulse**.

$F = \frac{(mv - mu)}{t}$ i.e. Force is a rate of change of momentum.

From this we can derive Newton's Second Law:

$F = m\frac{(v-u)}{t}$, and we know $a = \frac{(v-u)}{t}$ hence F = ma.

$F = \frac{(mv - mu)}{t}$ **s**hows us why we bend our legs when landing, why we have crumple zones in cars, seatbelts stretch and how we catch cricket balls etc. In all of these examples, we are trying to **increase the time period**, which will result in a smaller force. Conversely, karate hits are meant to be quick and sharp to decrease the time of contact, thus increasing the force applied.

An applied force means a change in momentum, but this does not necessarily mean a change in speed (as momentum is a vector quantity).

Conservation of Momentum

Momentum is conserved in every system (from quantum to astronomical), and including angular momentum (which is why it is hard to fall off a moving bicycle, and why gyroscopes work etc.).

Momentum before a collision is the same as momentum after the collision:

$$\Sigma \text{ all momentums before} = \Sigma \text{ all momentums after.}$$

Collisions are either:

Elastic – KE is conserved as well as momentum. This happens rarely in the macroscopic world, but frequently in the microscopic. Snooker balls and Newton's cradle are approximately elastic.

Inelastic – KE is not conserved, but momentum is. This is the one where two colliding bodies 'stick together' (and stop completely if they are in opposing directions) – a usual, everyday collision.

Remember that direction of momentums are important!

Jets, rockets and helicopters all work because of momentum. In the case of rockets, the fuel is forced out of the back at high velocity (and ignited). This results in an equal and opposite reaction (Newton's Third Law) on the rocket. Since the rocket is a much larger mass than the fuel, the fuel needs to have a large velocity to have a conservation of momentum.

When a rocket takes off, it appears to be slow. The thrust stays constant, but it gets faster as it gets higher because as the fuel burns, the mass of the rocket decreases, resulting in a larger acceleration.